ON THE COHEN-MACAULAY AND GORENSTEIN PROPERTIES OF JACOBSON GRAPH OF COMMUTATIVE RINGS

ESHA NAIR. M

Abstract

For a finite commutative ring R with non zero identity, one can define the *Jacobson graph* denoted by $\mathfrak{J}(R)$ as the graph with vertex set $R \setminus J(R)$, and two distinct vertices x and y are adjacent if and only if $1 - xy$ is not a unit in R.

In this work, we study the Cohen–Macaulay and Gorenstein properties of the Jacobson graph $\mathfrak{J}(R)$ via combinatorial methods. Since these properties are difficult to verify directly, we use the notion of well-covered graphs as a key tool. A graph is said to be well-covered if all its maximal independent sets have the same cardinality.

We first analyze structural properties of $\mathfrak{J}(R)$ and determine conditions under which it is well-covered. Using these conditions, we obtain necessary and sufficient criteria for $\mathfrak{J}(R)$ to be Cohen-Macaulay. Further, we investigate when $\mathfrak{J}(R)$ satisfies the stronger Gorenstein property. The results are expressed in terms of the algebraic structure of R, particularly its decomposition into local rings.

These results give a classification of finite commutative rings whose Jacobson graphs satisfy these properties and establish a clear connection between ring-theoretic structure and homological properties of the associated graph.

Esha Nair. M, Department of Mathematics, Pondicherry University, Puducherry-605 014, India.

Email address: eshanairm1999@gmail.com